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A LOGARITHMIC MEAN TEMPERATURE PROFILE IN THERMAL TURBULENCE

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NOMENCLATURE

A, constant;
a, layer half depth;
B_1, B_2, constants;
C, constant;
D, exponent;
f_1, f_2, functions;
k, thermal conductivity;
q, heat flux;
r, correlation coefficient;
T, mean temperature;
T_L, non-dimensional mean temperature;
T_P, non-dimensional mean temperature;
Z, vertical coordinate;
Z_+, non-dimensional coordinate;
$\tilde{Z}$, non-dimensional coordinate;
ΔT, temperature difference across layer;
δ_T, conduction layer thickness;
Nu, Nusselt number;
Ra, Rayleigh number.

INTRODUCTION

THE PURPOSE of this note is to demonstrate the existence of a logarithmic region in the mean temperature profile of natural convection thermal turbulence. The flow under consideration is generated between parallel horizontal surfaces by uniformly heating the lower boundary and uniformly cooling the upper boundary so as to establish a statistically steady turbulence having no mean velocity. Many experiments have investigated thermal turbulence, and mean temperature profiles have been reported in [1–4] and other papers. Formulas for the profile have been proposed by Priestly [5], Malkus [6], Kraichnan [7], and Howard [8]. Each suggested power laws of the form

$$T_P = CZ^D, \tag{1}$$

where

$$T_P = \frac{T(Z)-T(a)}{\Delta T/2}$$

and Z is the vertical distance from the boundary. The exponent D has been predicted to be either $-1/3$ or -1 depending in the fluid and/or region of the profile.

ANALYSIS

The following argument for the mean temperature profile in thermal turbulence is based on the derivation presented by Millikan [9] for the mean velocity profile in turbulent pipe flow. Consider a fluid layer of thickness $2a$ with the origin at the lower surface. By the definition of the Nusselt number one has

$$q = Nu\frac{k\Delta T}{2a}.$$

Experiments [1–4] show that in thermal turbulence most of the temperature drop occurs in conduction layers of thickness δ_T near the upper and lower surfaces

$$q = \frac{k\Delta T}{2\delta_T}.$$

It follows that

$$\delta_T = \frac{a}{Nu}. \tag{2}$$

Near the wall, δ_T is the characteristic length scale of the mean temperature profile. Therefore

$$T_L = f_1(Z_+)$$
$$\frac{dT_L}{dZ} = \frac{1}{\delta_T}\frac{df_1}{dZ_+} \tag{3}$$

where

$$Z_+ = \frac{Z}{\delta_T}$$

and

$$T_L = \frac{T(0)-T(Z_+)}{\Delta T}.$$

In the core region a is the appropriate scale. Therefore

$$T_L = f_2(\tilde{Z})$$
$$\frac{dT_L}{dZ} = \frac{1}{a}\frac{df_2}{d\tilde{Z}} \tag{4}$$

where

$$\tilde{Z} = \frac{Z}{a}.$$

Assuming that as Ra and Nu become very large there exists an overlap region where (3) and (4) are both valid, they can be equated to yield

$$Z_+\frac{df_1}{dZ_+} = \tilde{Z}\frac{df_2}{d\tilde{Z}} = A. \tag{5}$$

In equation (5) the variables have been separated. Integration produces

$$T_L(Z_+) = A\ln(Z_+)+B_1 \tag{6}$$

$$T_L(\tilde{Z}) = A\ln(\tilde{Z})+B_2. \tag{7}$$

An obvious transformation leads to a logarithmic law near the upper boundary.

If this argument is valid, (6) and (7) should hold in the same regions: near, but not adjacent to the horizontal boundaries in highly turbulent natural convection. By equating (6) and (7) in their region of common validity, the relationship between B_1 and B_2 is found to be

$$B_2 = B_1 + A\ln(Nu). \tag{8}$$

The assumption that δ_T and a are the only relevant length scales for the mean temperature profile is crucial to this development. If a mean flow were present there would be an additional scale—the viscous sublayer thickness—and

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Table 1. Comparison of logarithmic and power law fits

Set.	Ref.	Fluid	Ra	Aspect ratio	Range of fit from $Z_+ =$	Range of fit to $Z_+ =$	Logarithmic law r	Logarithmic law A	Logarithmic law B_1	Power law r	Power law D
1	[1]	air	8.9×10^5	5	2·2	5·38	0·9967	0·0258	0·4500	−0·9743	−1·672
2*	[1]	air	9×10^6 to 2×10^8	2 and 1	2·0	4·05	0·9914	0·0347	0·4320	−0·9871	−1·134
3	[2]	air	7.8×10^7	2·7	1·86	4·62	0·9950	0·0544	0·4006	−0·9920	−1·438
4	[3]	air	10^7	5·2	2·56	5·84	0·9961	0·0296	0·4410	−0·9920	−1·849
5	[3]	air	6.3×10^5	9·5	1·15	2·88	0·9959	0·067	0·432	−0·8971	−4·78
6	[4]	water	1.86×10^7	1·2	0·459	1·11	0·9996	0·225	0·376	−0·9940	−1·22

*Data was digitized from a smooth curve which fit several experimental runs.

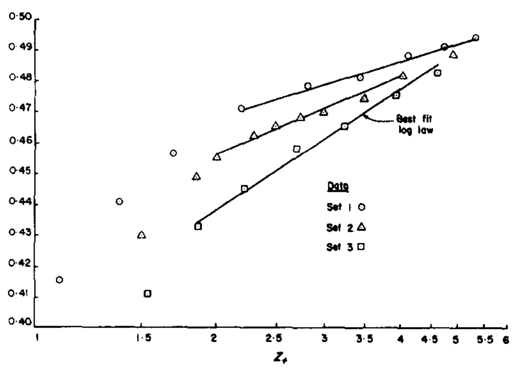

FIG. 1. Comparison of logarithmic mean temperature law with data.

this argument would not apply. If the mean flow is sufficiently strong, the conduction layer thickness is no longer important, and the forced convection case occurs. It is remarkable that in certain cases of forced convection, the mean temperature follows another logarithmic law [10].

A modification of the similarity argument used by Landau and Lifshitz [11] to derive the logarithmic velocity profile also applies to the present case.

COMPARISON WITH EXPERIMENTS

In order to assess the validity of the logarithmic law (6), data were digitized from those published graphs which were detailed enough to reveal a log region. When possible, measured data points were chosen; in some cases points were randomly selected from smooth curves. Six sets of data were examined. Table 1 shows the results obtained by fitting each set with logarithmic and power laws by the method of least squares. In every case there is a region for which the log law is a better representation than any power law. Furthermore, the best fit power law is never in accord with any of the theories. Compared to $-1/3$ or -1 laws, the advantage of the log law would be more dramatic. Figure 1 shows the best fit log laws for Sets 1, 2, and 3.

At present there is not sufficient data to determine universal values for A, B_1, and the limits of the log region. Sets 1–5 all deal with air, but the relatively low Rayleigh numbers and aspect ratios (minimum horizontal dimension divided by depth) probably account for the scatter in these values. In particular, at higher Rayleigh numbers the extent of the log region should be greater.

Set 6, which is for water, has values of A, B_1, and range of fit which are quite different from those for air. This is probably a Prandtl number effect, but more data will be needed to unravel its nature.

Thomas and Townsend [12] in 1957 suggested that the existence of a log region in the temperature profile was evidence of a strong mean flow. They obtained a highly asymmetric profile in air for $Ra = 6.75 \times 10^5$ and aspect ratio = 5·1. Near the boundaries they detected logarithmic regions which they attributed to fully developed forced convection thermal boundary layers. This would imply a Reynolds number of at least 4×10^5 [13]. Assuming that the characteristic length is the width of the apparatus (40 cm) and that the mean air temperature was 30°C, the required mean velocity would be 1600 cm/s. It is certain that this did not occur. Moreover, Thomas and Townsend's log region lies far from that of other experiments. Therefore it seems that their result was anomalous and that the existence of a logarithmic region *is* characteristic of thermal turbulence.

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